

## WEEKLY TEST OYJ SOLUTION 15 JULY 2019

### MATHEMATICS

31. (d) Let  $f(x) = 2x^3 - 24x + 107$   
 At  $x = -3$ ,  $f(-3) = 2(-3)^3 - 24(-3) + 107 = 125$   
 At  $x = 3$ ,  $f(3) = 2(3)^3 - 24(3) + 107 = 89$   
 For maxima or minima,  $f'(x) = 6x^2 - 24 = 0$   
 $\Rightarrow x = 2, -2$   
 So at  $x = 2$ ,  $f(2) = 2(2)^3 - 24(2) + 107 = 75$   
 at  $x = -2$ ,  $f(-2) = 2(-2)^3 - 24(-2) + 107 = 139$   
 Thus the maximum value of the given function in  $[-3, 3]$  is 139.

32. (c) Given  $f(x) = \frac{[(5+x)(2+x)]}{[1+x]}$   
 $f(x) = 1 + \frac{4}{1+x} + (5+x) = (6+x) + \frac{4}{(1+x)}$   
 $\Rightarrow f'(x) = 1 - \frac{4}{(1+x)^2} = 0$ ;  $x^2 + 2x - 3 = 0 \Rightarrow x = -3, 1$   
 Now  $f''(x) = \frac{8}{(1+x)^3}$ ,  $f''(-3) = -ve$ ,  $f''(1) = +ve$   
 Hence minimum value at  $x = 1$   
 $f(1) = \frac{(5+1)(2+1)}{(1+1)} = \frac{6 \times 3}{2} = 9$ .

33. (c)  $a^2x^4 + b^2y^4 = c^6 \Rightarrow y = \left( \frac{c^6 - a^2x^4}{b^2} \right)^{1/4}$   
 Hence  $f(x) = xy = x \left( \frac{c^6 - a^2x^4}{b^2} \right)^{1/4}$   
 $\Rightarrow f(x) = \left( \frac{c^6x^4 - a^2x^8}{b^2} \right)^{1/4}$   
 Differentiate  $f(x)$  with respect to  $x$ , then  
 $f'(x) = \frac{1}{4} \left( \frac{c^6x^4 - a^2x^8}{b^2} \right)^{-3/4} \left( \frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2} \right)$   
 Put  $f'(x) = 0$ ,  $\frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2} = 0$   
 $\Rightarrow x^4 = \frac{c^6}{2a^2} \Rightarrow x = \pm \frac{c^{3/2}}{2^{1/4}\sqrt{a}}$   
 At  $x = \frac{c^{3/2}}{2^{1/4}\sqrt{a}}$  the  $f(x)$  will be maximum, so  
 $f\left(\frac{c^{3/2}}{2^{1/4}\sqrt{a}}\right) = \left( \frac{c^{12}}{2a^2b^2} - \frac{c^{12}}{4a^2b^2} \right)^{1/4} = \left( \frac{c^{12}}{4a^2b^2} \right)^{1/4} = \frac{c^3}{\sqrt{2ab}}$ .

34. (b)  $y = f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve  $f'(x) = -3x^2 + 6x + 9$

Let  $g(x) = f'(x) = -3x^2 + 6x + 9$

Differentiate with respect to  $x$ ,  $g'(x) = -6x + 6$

Put  $g'(x) = 0 \Rightarrow x = 1$

Now,  $g''(x) = -6 < 0$  and hence at  $x = 1$ ,  $g(x)$  (slope) will have maximum value.

$\therefore [g(1)]_{\max.} = -3 \times 1 + 6 + 9 = 12.$

35. (d)  $f(x) = |px - 9| + r|x|, x \in (-\infty, \infty)$

Where  $p > 0, q > 0$  and  $r > 0$  can assume its minimum value only at one point, if  $p = q = r.$

36. (a) Let co-ordinate of  $R(x, 0)$

Given  $P(1, 1)$  and  $Q(3, 2)$

$$PR + RQ = \sqrt{(x-1)^2 + (0-1)^2} + \sqrt{(x-3)^2 + (0-2)^2}$$

$$= \sqrt{x^2 - 2x + 2} + \sqrt{x^2 - 6x + 13}$$

For minimum value of  $PR + RQ, \frac{d}{dx}(PR + RQ) = 0$

$$\Rightarrow \frac{d}{dx}(\sqrt{x^2 - 2x + 2}) + \frac{d}{dx}(\sqrt{x^2 - 6x + 13}) = 0$$

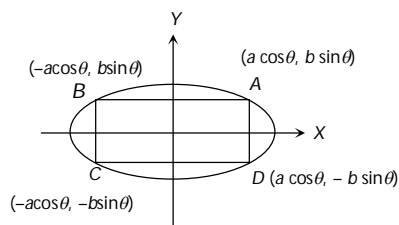
$$\Rightarrow \frac{(x-1)}{\sqrt{x^2 - 2x + 2}} = -\frac{(x-3)}{\sqrt{x^2 - 6x + 13}}$$

Squaring both sides,  $\frac{(x-1)^2}{(x^2 - 2x + 2)} = \frac{(x-3)^2}{x^2 - 6x + 13}$

$$\Rightarrow 3x^2 - 2x - 5 = 0 \Rightarrow (3x-5)(x+1) = 0, x = \frac{5}{3}, -1.$$

Also  $1 < x < 3. \therefore R = (5/3, 0).$

37. (c)



Area of rectangle  $ABCD$

$$= (2a \cos \theta) \cdot (2b \sin \theta) = 2ab \sin 2\theta$$

Hence, area of greatest rectangle is equal to  $2ab$ , when  $\sin 2\theta = 1.$

38. (b) To be increasing  $f'(x) = 3x^2 - 27 > 0$

$$\Rightarrow x^2 > 9 \Rightarrow |x| > 3.$$

39. (b) Since  $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$  is decreasing for all real values of  $x$ , therefore  $f'(x) < 0$  for all  $x$ .

$$\Rightarrow \sqrt{3} \cos x + \sin x - 2a < 0 \text{ for all } x$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x < a \text{ for all } x$$

$$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a \text{ for all } x \Rightarrow a \geq 1, \left[ \because \sin\left(x + \frac{\pi}{3}\right) \leq 1 \right].$$

40. (c) The function  $f(x) = x^3$  increases for all  $x$  and the function  $g(x) = 6x^2 + 15x + 5$  increases, if

$$g'(x) > 0 \Rightarrow 12x + 15 > 0 \Rightarrow x > -\frac{5}{4}.$$

Thus  $f(x)$  and  $g(x)$  both increases for  $x > -\frac{5}{4}.$

It is given that  $f(x)$  increases less rapidly than  $g(x)$ ,

Therefore the function  $\phi(x) = f(x) - g(x)$  is decreasing function, which implies that  $\phi'(x) < 0$

$$\Rightarrow 3x^2 - 12x - 15 < 0 \Rightarrow -1 < x < 5.$$

41. (d) If  $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$  decreases monotonically for all  $x \in R$ , then  $f'(x) \leq 0$  for all  $x \in R$

$$\Rightarrow 3(a+2)x^2 - 6ax + 9a \leq 0 \text{ for all } x \in R$$

$$\Rightarrow (a+2)x^2 - 2ax + 3a \leq 0 \text{ for all } x \in R$$

$$\Rightarrow a+2 < 0 \text{ and Discriminant} \leq 0$$

$$\Rightarrow a < -2, -8a^2 - 24a \leq 0 \Rightarrow a < -2 \text{ and } a(a+3) \geq 0$$

$$\Rightarrow a < -2, a \leq -3 \text{ or } a \geq 0 \Rightarrow a \leq -3 \Rightarrow -\infty < a \leq -3.$$

42. (c)  $f(x) = \frac{\sin x - x \cos x}{\sin^2 x} = \frac{\cos x(\tan x - x)}{\sin^2 x}$

$$0 < x \leq 1 \Rightarrow x \in Q_1 \Rightarrow \tan x > x, \cos x > 0$$

$$\therefore f(x) > 0 \text{ for } 0 < x \leq 1$$

$\therefore f(x)$  is an increasing function.

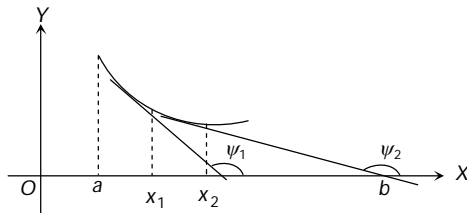
$$g(x) = \frac{\tan x - x \sec^2 x}{\tan^2 x} = \frac{\sin x \cos x - x}{\sin^2 x} = \frac{\sin 2x - 2x}{2 \sin^2 x}$$

$$(\sin 2x - 2x)' = 2 \cos 2x - 2 = 2[\cos 2x - 1] < 0$$

$$\Rightarrow \sin 2x - 2x \text{ is decreasing} \Rightarrow \sin 2x - 2x < 0$$

$$\therefore g'(x) < 0 \Rightarrow g(x) \text{ is decreasing.}$$

43. (d) From the trend of value of  $\sin x$  and  $\cos x$  we know  $\sin x$  and  $\cos x$  decrease in  $\frac{\pi}{2} < x < \pi$ . So, the statement S is correct.



The statement R is incorrect which is clear from graph. Clearly  $f(x)$  is differentiable in  $(a, b)$ .

Also,  $a < x_1 < x_2 < b$ .

$$\text{But } f'(x_1) = \tan \phi_1 < \tan \phi_2 = f'(x_2).$$

44. (d) Given  $f(x) = x^3 + bx^2 + cx + d$

$$\therefore f'(x) = 3x^2 + 2bx + c$$

$$\text{Now its discriminant} = 4(b^2 - 3c)$$

$$\Rightarrow 4(b^2 - c) - 8c < 0, \text{ as } b^2 < c \text{ and } c > 0$$

Therefore,  $f'(x) > 0$  for all  $x \in R$

Hence  $f$  is strictly increasing.

45. (a)  $f(x) = 3x^2 - 2x + 1, f'(x) = 6x - 2 \geq 0 \Rightarrow x \geq \frac{1}{3}$

Option (a) is incorrect. Checking other function similarly we find that they are correctly matched